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PHYSICS ACADEMY

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CSIR-NET-JRF

CLASSICAL MECHANICS

PREVIOUS YEAR QUESTIONS WITH ANSWER (CHAPTER-WISE)

- **Newtonian Mechanics**
- **Central Force**
- **Special Theory of Relativity**
- **Lagrangian**
- **Hamiltonian**
- **Poisson Bracket & Canonical Transformation**
- **Phase Space Trajectory**

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14, RAIPUR ROAD, NEAR SURVEY CHOWK, DEHRADUN CONTACT:- +919045460409, +919870827730 website:- www.careerspectra.in

NEWTONIAN MECHANICS

1. A particle of unit mass moves in a potential $V(x) = ax^2 + \frac{b}{x^2}$, where a and b are positive constants. The angular frequency of small oscillations about the minimum of the potential is. **[CSIR-JUNE-2011]**

(a) $\sqrt{8b}$ (b) $\sqrt{8a}$ (c) $\sqrt{8a/b}$ (c) $\sqrt{8b/a}$

2. An annulus of mass M made of a material of uniform density has inner and outer radii a and b respectively. Its principle moment of inertia along the axis of symmetry perpendicular to the plane of the annulus is: **[CSIR-DEC-2011]**

(a) $\frac{1}{2}M\frac{(b^4+a^4)}{(b^2-a^2)}$ (b) $\frac{1}{2}M\pi(b^2-a^2)$

(c) $\frac{1}{2}M(b^2-a^2)$ (d) $\frac{1}{2}M(b^2+a^2)$

3. A horizontal circular platform mutes with a constant angular velocity Ω directed vertically upwards. A person seated at the center shoots a bullet of mass m horizontally with speed v. The acceleration of the bullet, in the reference frame of the shooter, is **[CSIR-JUNE-2012]**

(a) $2v\,\Omega$ to his right (b) $2v\,\Omega$ to his left

(c) $v\,\Omega$ to his right (d) $v\,\Omega$ to his left

4. Consider the motion of a classical particle in a one dimensional double-well Potential V(x) = $\frac{1}{4}(x^2-2)^2$. If the particle is displaced infinitesimally from the minimum on the x-axis (and friction is neglected), then **[CSIR-JUNE-2012]**

(a) The particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$.

(b) The particle will execute simple harmonic motion in the right well with an angular Frequency $\omega = 2$

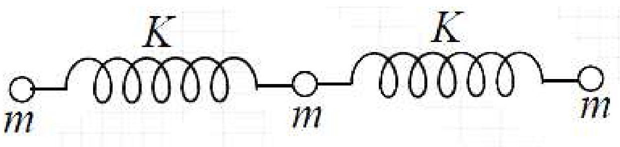
(c) The particle will switch between the right and left wells

(d) The particle will approach the bottom of the right well and settle there

5. A solid cylinder of height H, radius R and density ρ, floats vertically on the surface of a liquid of density ρ_0. The cylinder will be set into oscillatory motion when a small instantaneous downward force is applied. The frequency of oscillation is. **[CSIR-DEC-2012]**

(a) $\frac{\rho g}{\rho_0 H}$ (b) $\frac{\rho}{\rho_0}\sqrt{\frac{g}{H}}$ (c) $\sqrt{\frac{\rho g}{\rho_0 H}}$ (d) $\sqrt{\frac{\rho_0 g}{\rho H}}$

6. Three particles of equal mass (m) are connected by two identical massless springs of stiffness constant (K) as shown in the figure **[CSIR-DEC-2012]**



If x_1, x_2 and x_3 denote the horizontal displacement of the masses from their respective equilibrium positions the potential energy of the system is.

(a) $\frac{1}{2}K[x_1^2 + x_2^2 + x_3^2]$

(b) $\frac{1}{2}K[x_1^2 + x_2^2 + x_3^2 - x_2(x_1 + x_3)]$

(c) $\frac{1}{2}K[x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_3)]$

(b) $\frac{1}{2}K[x_1^2 + 2x_2^2 - 2x_2(x_1 + x_3)]$

7. Two bodies of equal mass m are connected by a massless rigid rod of length l lying in the xy-plane with the center of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is.

[CSIR-DEC-2012]

(a) $ml^2\omega/4$ (b) $ml^2\omega/2$ (c) $ml^2\omega$ (d) $2ml^2\omega$

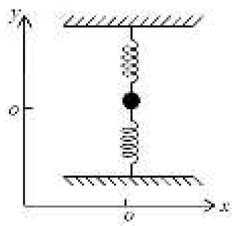
8. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is. **[CSIR-DEC-2013]**

(a) $2\pi\sqrt{\frac{l+R}{g}}$ (b) $\frac{2\pi}{\sqrt{g}}(l^2 + R^2)^{1/4}$

(c) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+1)}}$ (d) $\frac{2\pi}{\sqrt{g}}(2R^2 + 2Rl + l^2)^{1/4}$

9. Consider a particle of mass m attached to two identical springs each of length l and spring constant k (see the figure). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations?

[CSIR-DEC-2013]



(a) $m\ddot{x} + \frac{kx^3}{l^2} = 0$ (b) $m\ddot{x} + kx = 0$

(a) $m\ddot{x} + 2kx = 0$ (d) $m\ddot{x} + \frac{kx^3}{l} = 0$

10. A particle of mass m is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^2$ where a, b are positive constants. The frequency of small oscillations about a point of stable equilibrium is. **[CSIR-DEC-2014]**

(a) $\sqrt{a/m}$ (b) $\sqrt{2a/m}$ (c) $\sqrt{3a/m}$ (d) $\sqrt{6a/m}$

11. The radius of Earth is approximately 6400 km. The height h at which the acceleration due to Earth's gravity differs from g at the Earth's surface by approximately 1% is. **[CSIR-DEC-2014]**

(a) 64 km (b) 48 km (c) 32 km (d) 16 km

12. A particle moves in two dimensions on the ellipse $x^2 + 4y^2 = 8$. At a particular instant it is at the point (x,y) = (2,1) and the x -component of its velocity is 6 (in suitable units). Then the y -component of its velocity is. **[CSIR-JUNE-2015]**

(a) -3 (b) -2 (c) 1 (d) 4

13. A particle of mass m moves in the one-dimensional potential $V(x) = \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$ where $\alpha, \beta > 0$. One of the equilibrium points is $x = 0$. The angular frequency of small oscillations about the other equilibrium point is. **[CSIR-JUNE-2015]**

(a) $\frac{2\alpha}{\sqrt{3m\beta}}$ (b) $\frac{\alpha}{\sqrt{m\beta}}$ (a) $\frac{\alpha}{\sqrt{12m\beta}}$ (a) $\frac{\alpha}{\sqrt{24m\beta}}$

14. A particle of unit mass moves in the xy -plane in such a way that $\dot{x}(t) = y(t)$ and $\dot{y}(t) = -x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential. **[CSIR-JUNE-2015]**

(a) $\frac{1}{2}(x^2 + y^2)$ (b) $\frac{1}{2}(x^2 - y^2)$

(c) $x + y$ (d) $x - y$

15. Two masses m each, are placed at the points $(x, y) = (a, a)$ and $(-a, a)$ and two masses, $2m$ each, are placed at the points $(a, -a)$ and $(-a, a)$. The principal moments of inertia of the system are. **[CSIR-DEC-2015]**

(a) $2m^2, 4ma^2$ (b) $4ma^2, 8ma^2$

(c) $4ma^2, 4ma^2$ (d) $8ma^2, 8ma^2$

16. A ball of mass m , initially at rest, is dropped from a height of 5 meters. If the coefficient of restitution is 0.9 , the speed of the ball just before it hits the floor the second time is approximately (take g = 9.8 m/s^2). **[CSIR-JUNE-2016]**

(a) 9.80 m/s (b)9.10 m/s

(c)8.91 m/s (d) 7.02 m/s

17. For a particle of energy E and momentum p (in a frame F), the rapidity y is defined As $y = \frac{1}{2}\ln\left(\frac{E+p_3c}{E-p_3c}\right)$. In a frame F' moving with velocity $v = (0,0,\beta c)$ with respect to F , the rapidity y' will be. **[CSIR-JUNE-2016]**

(a) $y' = y + \frac{1}{2}\ln(1-\beta^2)$ (b) $y' = y + \frac{1}{2}\left(\frac{1+\beta}{1-\beta}\right)$

(c) $y' = y + \ln\left(\frac{1+\beta}{1-\beta}\right)$ (d) $y' = y + 2\ln\left(\frac{1+\beta}{1-\beta}\right)$

18. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the ball experiences a damping force of the form $-\gamma v$, where v is its instantaneous velocity and γ is a constant. Given the values m = 10kg, $\gamma = 10kg/s$ and $g \approx 10m/s^2$ the distance travelled (in meters) in time t in seconds, is. **[CSIR-DEC-2016]**

(a) $10(t+1-e^{-t})$ (b) $10(t-1+e^{-t})$

(c) $5t^2-(1-e^{t})$ (d) $5t^2$

19. A particle in two dimensions is in a potential V(x,y) = x +2y. Which of the following (apart from the total energy of the particle) is also a constant of motion? **[CSIR-DEC-2016]**

(a) $p_y - 2p_x$ (b) $p_x - 2p_y$

(c) $p_x + 2p_y$ (d) $p_y + 2p_x$

20. A ball weighing 100 *gm*, released from a height of 5*m*, bounces perfectly elastically off a plate. The collision time between the ball and the plate is 0.5 *s* . The average force on the plate is approximately. **[CSIR-JUNE-2017]**

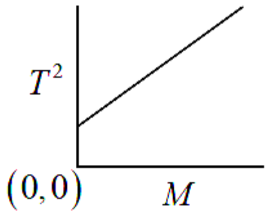
(a) 3*N* (b) 2 *N* (c) 5*N* (d) 4 *N*

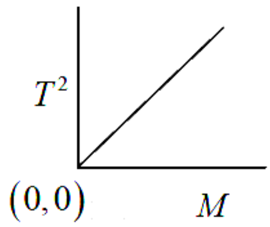
21. A solid vertical rod, of length *L* and cross-sectional area *A*, is made of a material of Young's modulus *Y*. The rod is loaded with a mass *M*, and, as a result, extends by a small amount Δ *L* in the equilibrium condition. The mass is then suddenly reduced to *M* / 2. As a result, the rod will undergo longitudinal oscillation with an angular frequency. **[CSIR-JUNE-2017]**

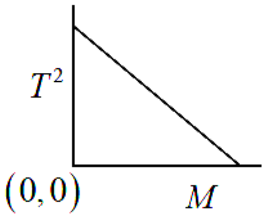
(a) $\sqrt{2YA/ML}$ (b) $\sqrt{YA/ML}$

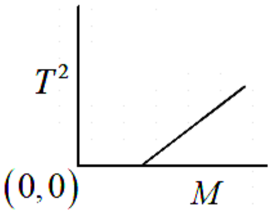
(c) $\sqrt{2YA/M\Delta L}$ (d) $\sqrt{YA/M\Delta L}$

22. The spring constant *k* of a spring of mass *s m* is determined experimentally by loading the spring with mass *M* and recording the time period *T*, for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is. **[CSIR-DEC-2017]**

(a)


(b)


(c)


(d)


23. Consider a set of particles which interact by a pair potential $V = ar^6$ where r is the interparticle separation and $a > 0$ is a constant. If a system of such particles has reached virial equilibrium, the ratio of the kinetic to the total energy of the system is. **[CSIR-DEC-2017]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

24. A particle moves in the one-dimensional potential $V(x) = \alpha x^6$, where $a > 0$ is a constant. If the total energy of the particle is E, its time period in a periodic motion is proportional to **[CSIR-JUNE-2018]**

(a) $E^{-1/3}$ (b) $E^{-1/2}$ (c) $E^{1/3}$ (d) $E^{1/2}$

25. A particle of mass m, kept in potential $V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4$ (where k and λ are positive constants), undergoes small oscillations about an equilibrium point. The frequency of oscillations is. **[CSIR-JUNE-2018]**

(a) $\frac{1}{2\pi}\sqrt{\frac{2\lambda}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (a) $\frac{1}{2\pi}\sqrt{\frac{\lambda}{m}}$

26. A particle of mass m moves in a central potential $V(r) = -\frac{k}{r}$ in an elliptic orbit $r(\theta) = \frac{a(1-e^2)}{1+e\cos\theta}$, where $0 \leq \theta < 2\pi$ and a and e denote the semi-major axis and eccentricity, respectively. If its total energy is $E = -\frac{k}{2a}$, the maximum kinetic energy is. **[CSIR-JUNE-2018]**

(a) $E(1-e^2)$ (b) $E\,\frac{(e+1)}{(e-1)}$

(c) $E/(1-e^2)$ (b) $E\,\frac{(e-1)}{(e+1)}$

27. A particle of mass m, moving along the x - direction, experiences a damping force $-\gamma v^2$, where γ is a constant and v is its instantaneous speed. If the speed at t = 0 is v_0, the speed at time t is. **[CSIR-DEC-2018]**

(a) $v_0 e^{-\frac{\gamma v_0 t}{m}}$
(b) $\frac{v_0}{1+1n\left(1+\frac{\gamma v_0 t}{m}\right)}$
(c) $\frac{m v_0}{m+\gamma v_0 t}$
(d) $\frac{2v_0}{1+e^{\frac{\gamma v_0 t}{m}}}$

28. The time period of a particle of mass m, undergoing small oscillations around x = 0 , in the potential $V = V_0 \cos h\left(\frac{x}{L}\right)$, is. **[CSIR-DEC-2018]**

(a) $\pi\sqrt{\frac{mL^2}{V_0}}$
(b) $2\pi\sqrt{\frac{mL^2}{2V_0}}$
(c) $2\pi\sqrt{\frac{mL^2}{V_0}}$
(d) $2\pi\sqrt{\frac{2mL^2}{V_0}}$

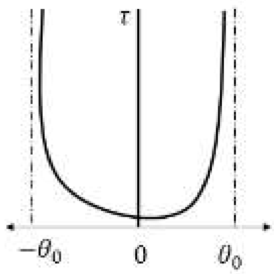
29. A point mass m is constrained to move on the inner surface of a paraboloid of revolution $x^2+y^2=az$ (where a > 0 is a constant). When it spirals down the surface under the influence of gravity (along –z direction). The angular speed about the z-axis is proportional to. **[CSIR-NOV-2020]**

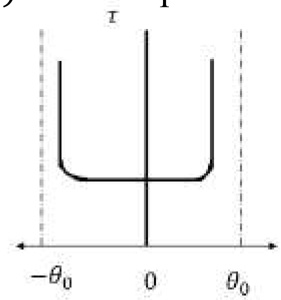
(a) 1 (independent of z)
(b) z
(c) z^{-1}
(d) z^{-2}

30. Falling drops of rain breaks up and coalesce with each other and finally achieve an approximately spherical shape in the steady state. The radius of such a drop scales with the surface tension a as. **[CSIR-NOV-2020]**

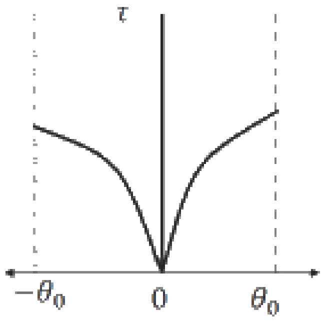
(a) $1/\sqrt{\sigma}$
(b) $\sqrt{\sigma}$
(c) σ
(d) σ^2

31. A pendulum executes small oscillations between angles $+\theta_0$ and $-\theta_0$. If $r(\theta)d\theta$ is the time spent between θ and $d\theta$, then $t(\theta)$ is best represented by. **[CSIR-NOV-2020]**

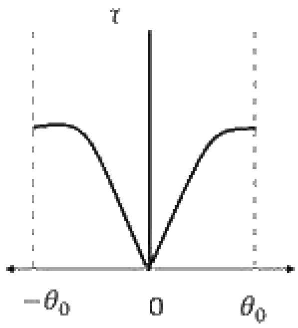
(a)


(b)


(c)



(d)



32. The velocity $v(x)$ of a particle moving in one dimension is given by $v(x) = v_0 \sin\left(\frac{ax}{x_0}\right)$, where v_0 and x_0 are positive constants of appropriate dimensions. If the particle is initially at $x/x_0 = \epsilon$. Where $|\epsilon| \ll 1$. Then, in the long time.it **[CSIR-NOV-2020]**

(a) Executes an oscillatory motion around x = 0
(b) Tends towards x = 0
(c) Tends towards x = x_0
(d) Executes an oscillatory motion around x = x_0

33. A rod pivoted at one end is rotating clock wise 25 times a second in a plane. A video camera which records at a rate of 30 frames per second is used to film. The motion to someone watching the video the apparent motion of the rod will seem to be. **[CSIR-NOV-2020]**

(a) 10 rotations per second in the clockwise direction.
(b) 10 rotations per second in the anti-clockwise direction.
(c) 5 rotations per second in the clockwise direction.
(d) 5 rotations per second in the anti-clockwise direction.

CENTRAL FORCE

1. The acceleration due to gravity (*g*) on the surface of Earth is approximately 2.6 times that on the surface of Mars. Given that the radius of Mars is about one half the radius of Earth, the ratio of the escape velocity on Earth to that on Mars is approximately. **[CSIR-JUNE-2011]**

(a) 1.1 (b) 1.3 (c) 2.3 (d) 5.2

2. The potential of a diatomic molecule as a function of the distance *r* between the atoms is given by $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. The value of the potential at equilibrium separation between the atoms is: **[CSIR-DEC-2011]**

(a) $-4a^2/b$ (b) $-2a^2/b$ (c) $-a^2/2b$ (d) $-a^2/4b$

3. Two particles of identical mass move in circular orbits under a central potential $V(r) = \frac{1}{2}kr^2$. Let l_1 and l_2 be the angular momenta and r_1, r_2 be the radii of the orbits respectively. If $\frac{l_1}{l_2} = 2$, the value of $\frac{r_1}{r_2}$ is: **[CSIR-DEC-2011]**

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$ (c) 2 (d) ½

4. A planet of mass *m* moves in the inverse square central force field of the Sun of mass *M* . If the semi-major and semi-minor axes of the orbit are *a* and *b*, respectively, the total energy of the planet is: **[CSIR-DEC-2011]**

(a) $-\frac{GMm}{a+b}$ (b) $-GMm\left(\frac{1}{a}+\frac{1}{b}\right)$

(c) $-\frac{GMm}{a}\left(\frac{1}{b}-\frac{1}{a}\right)$ (d) $-GMm\left(\frac{a-b}{(a+b)^2}\right)$

5. A planet of mass *m* moves in the gravitational field of the Sun (mass *M*). If the semimajor and semi-minor axes of the orbit are *a* and *b* respectively, the angular momentum of the planet is. **[CSIR-DEC-2012]**

(a) $\sqrt{2GMm^2(a+b)}$ (b) $\sqrt{2GMm^2(a-b)}$

(c) $\sqrt{\frac{2GMm^2ab}{a-b}}$ (d) $\sqrt{\frac{2GMm^2ab}{a+b}}$

6. A planet of mass mand an angular momentum L moves in a circular orbit in a potential, V(r) = -k/r, where k is a constant, if it is slightly perturbed radially, the angular frequency of radial oscillations is. **[CSIR-JUNE-2013]**

(a) $mk^2/\sqrt{2}L^3$ (b) mk^2/L^3

(c) $\sqrt{2}mk^2/L^3$ (d) $\sqrt{3}mk^2/L^3$

7. The probe Mangalyaan was sent recently to explore the planet Mars. The inter-planetary part of the trajectory is approximately a half-ellipse with the Earth (at the time of launch), Sun and Mars (at the time the probe reaches the destination) forming the major axis. Assuming that the orbits of Earth and Mars are approximately circular with radii R_E and R_M, respectively, the velocity (with respect to the Sun) of the probe during its voyage when it is at a distance $r(R_E \ll r \ll R_M)$ from the Sun, neglecting the effect of Earth and Mars, is

[CSIR-DEC-2014]

(a) $\sqrt{2GM\frac{(R_E+R_M)}{r(R_E+R_M-r)}}$ (b) $\sqrt{2GM\frac{(R_E+R_M-r)}{r(R_E+R_M)}}$

(c) $\sqrt{2GM\frac{R_E}{rR_M}}$ (d) (a) $\sqrt{\frac{2GM}{r}}$

8. Consider circular orbits in a central force potential $V(r) = -\frac{k}{r^n}$, where k > 0 and 0 < n < 2. If the time period of a circular orbit of radius R is T_1 and that of radius 2R is T_2, then $\frac{T_2}{T_1}$. **[CSIR-DEC-2016]**

(a) $2^{\frac{n}{2}}$ (b) $2^{\frac{2}{3}n}$ (b) $2^{\frac{n}{2}+1}$ (d) 2^n

9. In the attractive Kepler problem described by the central potential $V(r) = \frac{-k}{r}$ (where k is a positive constant), particle of mass m with a non-zero angular momentum can never reach the center due to the centrifugal barrier. If we modify the potential to. **[CSIR-DEC-2018]**

$$V(r) = -\frac{k}{r} - \frac{\beta}{r^3}$$

one finds that there is a critical value of the angular momentum ℓ_c below which there is no centrifugal barrier. This value of ℓ_c is.

(a) $[12km^2\beta]^{1/2}$ (b) $[12km^2\beta]^{-1/2}$
(c) $[12km^2\beta]^{1/4}$ (d) $[12km^2\beta]^{-1/4}$

10. A spacecraft of mass m = 1000kg has a fully reflecting sail that is oriented perpendicular to the direction of the sun. the sun radiates 10^{26}W and has a mass M = 10^{30} kg. Ignoring the effect of the planets. For the gravitational pull of the sun to balance the radiation pressure on the sail. The area of the sail will be. **[CSIR-NOV-2020]**

(a) 10^2m^2 (b) 10^4m^2 (c) 10^8m^2 (d) 10^6m^2

SPECIAL THEORY OF RELATIVITY

1. Consider the decay process $\tau^- \to \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of the τ^-, π^- and ν_τ are M_τ, M_π and zero respectively. **[CSIR-JUNE-2011]**

A. The energy of π^- is.

(a) $\frac{(M_\tau^2 - M_\tau^2)c^2}{2M_\tau}$ (b) $\frac{(M_\tau^2 + M_\tau^2)c^2}{2M_\tau}$ (c) $(M_\tau - M_\pi)c^2$ (d) $\sqrt{M_\tau M_\pi}c^2$

2. A constant force F is applied to a relativistic particle of rest mass m. If the particle starts from rest at $t = 0$, its speed after a time t is. **[CSIR-DEC-2011]**

(a) Ft/m (b) c tanh $\left(\frac{Ft}{mc}\right)$

(c) c $\left(1 - e^{-Ft/mc}\right)$ (d) $\frac{Fct}{\sqrt{F^2t^2+m^2c^2}}$

3. Two events separated by a (spatial) distance $9 \times 10^9 m$, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0.8 c (where the speed of light c = $3 \times 10^8 m/s$ is. **[CSIR-JUNE-2012]**

(a) 60s (b) 40s (c) 20 s (d) 0s

4. What is proper time interval between the occurrence of two events if in one inertial frame events are separated by 7.5×10^8 m and occur 6.5s apart? **[CSIR-JUNE-2012]**

(a) 6.50 s (b) 6.00 s (c) 5.75 s (d) 5.00 s

5. The muon has mass $105MeV/c^2$ and mean life time 2.2μs in its rest frame. The mean distance traversed by a muon of energy $315MeV$ before decaying is approximately, **[CSIR-DEC-2012]**

(a) $3\times 10^5 km$ (b) 2.2cm (c) 6.6μm (d) 1.98km.

6. The area of a disc in its rest frame S is equal to 1 (in some units). The disc will appear distorted to an observer O moving with a speed u with respect to S along the plane of the disc. The area of the disc measured in the rest frame of the observer O is (c is the speed of light in vacuum). **[CSIR-JUNE-2013]**

(a) $\left(1-\frac{u^2}{c^2}\right)^{1/2}$ (b) $\left(1-\frac{u^2}{c^2}\right)^{-1/2}$

(c) $\left(1-\frac{u^2}{c^2}\right)$ (d) $\left(1-\frac{u^2}{c^2}\right)^{1}$

7. The recently-discovered Higgs boson at the LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are 125 GeV/c^2 and 90 GeV/c^2 respectively, and the decaying Higgs particle is at rest, the energy of the photon will approximately be. **[CSIR-JUNE-2014]**

(a) $35\sqrt{3}\ GeV$ (b) $35GeV$ (c) 30GeV (d) 15GeV

8. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is: **[CSIR-DEC-2014]**

(a) $v = c\sqrt{1-\frac{mc^2}{E}}$ (b) $v=\sqrt{\frac{2E}{m}}\left(1+\frac{mc^2}{E}\right)$

(c) $v = c\sqrt{1-\left(\frac{mc^2}{E}\right)}$ (d) $v = c\left(1+\frac{mc^2}{E}\right)$

9. Consider three inertial frames of reference A, B and C. the frame B moves with a velocity $\frac{c}{2}$ with respect to A, and C moves with a velocity $\frac{c}{10}$ with respect to B in the same direction. The velocity of C as measured in A is. **[CSIR-JUNE-2015]**

(a) $\frac{3c}{7}$ (b) $\frac{4c}{7}$ (c) $\frac{c}{7}$ (d) $\frac{\sqrt{3}c}{7}$

10. Consider a particle of mass m moving with a speed v. If T_R denotes the relativistic kinetic energy and T_N its non-relativistic approximation, then the value of $\frac{(T_R - T_N)}{T_R}$ for $v = 0.01\,c$, is. **[CSIR-DEC-2015]**

(a) 1.25×10^{-5} (b) 5.0×10^{-5}
(c) 7.25×10^{-5} (d) 1.0×10^{-4}

11. Let (x, t) and (x', t') be the coordinate systems used by the observers O and O' respectively. Observer O' moves with a velocity $v = \beta c$ along their common positive x - axis. If $x_+ = x + ct$ and $x_- = x - ct$ are the linear combinations of the coordinates, the Lorentz transformation relating O and O' takes the form. **[CSIR-JUNE-2016]**

(a) $x'_+ = \frac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$ and $x'_- = \frac{x_- - \beta x_-}{\sqrt{1-\beta^2}}$

(b) $x'_+ = \sqrt{\frac{1+\beta}{1-\beta}}\,x_+$ and $x'_- = \sqrt{\frac{1-\beta}{1+\beta}}\,x_-$

(c) $x'_+ = \frac{x_+ - \beta x_-}{\sqrt{1-\beta^2}}$ and $x'_- = \frac{x_- - \beta x_+}{\sqrt{1-\beta^2}}$

(d) $x'_+ = \sqrt{\frac{1-\beta}{1+\beta}}\,x_+$ and $x'_- = \sqrt{\frac{1+\beta}{1-\beta}}\,x_-$

12. A relativistic particle moves with a constant velocity v with respect to the laboratory frame. In time τ, measured in the rest frame of the particle, the distance that it travels in the laboratory frame is. **[CSIR-DEC-2016]**

(a) $v\tau$ (b) $\frac{c\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ (c) $v\tau\sqrt{1-\frac{v^2}{c^2}}$ (d) $\frac{v\tau}{\sqrt{1-\frac{v^2}{c^2}}}$

13. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final, velocities (in the lab frame) is. **[CSIR-DEC-2016]**

(a) $\theta = \frac{\pi}{2}$ (b) $\theta = \pi$
(c) $0 < \theta \le \frac{\pi}{2}$ (d) $\frac{\pi}{2} < \theta \le \pi$

14. Consider a radioactive nucleus that is travelling at a speed $\frac{c}{2}$ with respect to the lab frame. It emits $\gamma -$ rays of frequency v_0 in its rest frame. There is a stationary detector, (which is not on the path of the nucleus) in the lab. If a $\gamma -$ ray photon is emitted when the nucleus is closest to the detector, its observed frequency at the detector is. **[CSIR-DEC-2016]**

(a) $\frac{\sqrt{3}}{2}v_0$ (b) $\frac{1}{\sqrt{3}}v_0$ (c) $\frac{1}{\sqrt{2}}v_0$ (d) $\sqrt{\frac{2}{3}}v_0$

15. An inertial observer sees two events E_1 and E_2 happening at the same location but $6\mu s$ apart in time. Another observer moving with a constant velocity *v* (with respect to the first one) sees the same events to be 9 μs apart. The spatial distance between the events, as measured by the second observer, is approximately. **[CSIR-JUNE-2017]**

(a) 300 m (b) 1000 m (c) 2000 m (d) 2700 m

16. A light signal travels from a point *A* to a point *B*, both within a glass slab that is moving with uniform velocity (in the same direction as the light) with speed 0.3*c* with respect to an external observer. If the refractive index of the slab is 1.5, then the observer will measure the speed of the signal as. **[CSIR-DEC-2017]**

(a) 0.67 *c* (b) 0.81*c* (c) 0.97 *c* (d) *c*

17. A cyclist, weighing a total of 80*kg* with the bicycle, pedals at a speed of 10 *m/ s*. She stops pedaling at an instant which is taken to be *t* = 0. Due to the velocity dependent frictional force, her velocity is found to vary as $v(t) = \frac{10}{\left(1+\frac{t}{30}\right)} ms,$ where *t* is measured in seconds. When the velocity drops to 8 *m/ s*, she starts pedaling again to maintain a constant speed. The energy expended by her in 1 minute at this (new) speed, is. **[CSIR-DEC-2017]**

(a) 4 *kJ* (b) 8 *kJ* (c) 16 *kJ* (d) 32 *kJ*

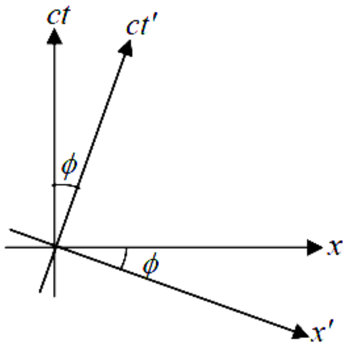
18. Two particles *A* and *B* move with relativistic velocities of equal magnitude *v*, but in opposite directions, along the *x* -axis of an inertial frame of reference. The magnitude of the velocity of *A*, as seen from the rest frame of *B*, is **[CSIR-JUNE-2018]**

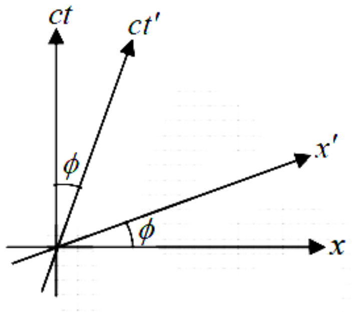
(a) $\frac{2v}{\left(1-\frac{v^2}{c^2}\right)}$ (b) $\frac{2v}{\left(1+\frac{v^2}{c^2}\right)}$ (c) $2v\sqrt{\frac{c-v}{c+v}}$ (d) $\frac{2v}{\sqrt{1-\frac{v^2}{c^2}}}$

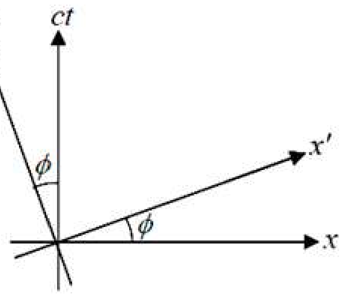
19. The energy of a free relativistic particle is $E = \sqrt{|\vec{p}|^2c^2 + m^2c^4}$, where *m* is its rest mass, $\vec{p}$ is its momentum and *c* is the speed of light in vacuum. The ratio v_g/v_p of the group v_g of a quantum mechanical wave packet (describing this particle) to the phase velocity v_p is. **[CSIR-JUNE-2018]**

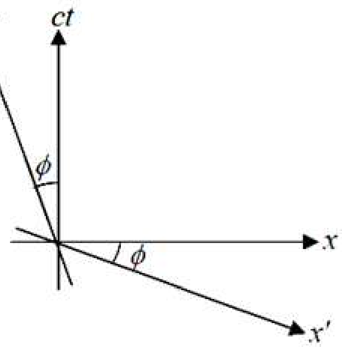
(a) $|\vec{p}|c/E$ (b) $|\vec{p}|mc^3/E^2$ (c) $|\vec{p}|\ c^3/E^2$ (d) $|\vec{p}|c/2E$

20. An inertial frame K' moves with a constant speed *v* with respect to another inertial frame *K* along their common *x* - direction. Let (x, ct) and (x', ct') denote the space time co-ordinates in the frames *K* and K', respectively. Which of the following space time diagrams correctly describes the $t' - axis$ $(x' = 0\ line)$ and the x'-axis $(t' = 0\ line)$ in the x-ct plane? (In the following figures tan $\phi = v/c$). **[CSIR-JUNE-2018]**

(a)


(c)


(c)


(d)


21. Consider the decay $A \to B + C$ of a relativistic spin $-\frac{1}{2}$ particles A. Which of the following statements is true in the rest frame of the particle A? **[CSIR-DEC-2018]**

(a) The spin of both B and C may be ½

(b) The sum of the masses of B and C is greater than the mass of A

(c) The energy of B is uniquely determined by the masses of the particles

(d) The spin of both B and C may be integral

22. A relativistic particle of mass m and charge e is moving in a uniform electric field of strength ε. Starting from rest at t = 0, how much time will it take to reach the speed $\frac{c}{2}$? **[CSIR-DEC-2018]**

(a) $\frac{1}{\sqrt{3}}\frac{mc}{e\varepsilon}$ (b) $\frac{mc}{e\varepsilon}$ (c) $\sqrt{2}\frac{mc}{e\varepsilon}$ (d) $\sqrt{\frac{3}{2}\frac{mc}{e\varepsilon}}$

23. A heavy particle of rest mass M while moving along the positive z-direction, decays into two identical light particles with rest mass m (where M > 2m). The maximum value of the momentum that any one of the lighter particles can have in a direction perpendicular to the z-direction, is **[CSIR-NOV-2020]**

(a) $\frac{1}{2}c\sqrt{M^2 - 4m^2}$ (b) $\frac{1}{2}c\sqrt{M^2 - 2m^2}$

(c) $c\sqrt{M^2 - 4m^2}$ (d) $\frac{1}{2}Mc$

LAGRANGIAN

1. The Lagrangian of a particle of charge e and mass m in applied electric and magnetic fields is given by $L = \frac{1}{2}m\vec{v}^2 + e\vec{A}.\vec{v} - e\phi$, where $\vec{A}$ and ϕ are the vector and scalar potentials corresponding to the magnetic and electric fields, respectively. Which of the following statements is correct?

[CSIR-JUNE-2011]

(a) The canonically conjugate momentum of the particle is given by $\vec{p} = m\vec{v}$

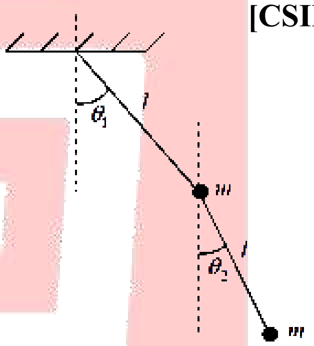
(b) The Hamiltonian of the particle is given by $H = \frac{\vec{p}^2}{2m} + \frac{e}{m}\vec{A}.\vec{p} + e\phi$.

(c) L remains unchanged under a gauge transformation of the potentials

(d) Under a gauge transformation of the potentials, L changes by the total time derivative

2. A double pendulum consists of two point masses m attached by strings of length l as shown in the figure: The kinetic energy of the pendulum is.

[CSIR-DEC-2011]



(a) $\frac{1}{2}ml^2[\dot{\theta}_1^2 + \dot{\theta}_2^2]$

(b) $\frac{1}{2}ml^2[2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)]$

(c) $\frac{1}{2}ml^2[\dot{\theta}_1^2 + 2\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)]$

(d) $\frac{1}{2}ml^2[2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 + \theta_2)]$

3. A particle of mass m moves inside a bowl. If the surface of the bowl is given by the Equation $z = \frac{1}{2}a(x^2 + y^2)$, where a is a constant, the Lagrangian of the particle is.

[CSIR-DEC-2011]

(a) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 - gar^2)$

(b) $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2]$

(c) $\frac{1}{2}m\ (\dot{r}^2 + r^2\dot{\phi}^2 + r^2 sin^2\theta\dot{\phi}^2 - gar^2)$

(d) $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2]$

4. If the Lagrangian of a particle moving in one dimension is given by L = $\frac{\dot{x}^2}{2x} - V(x)$ the Hamiltonian is.

[CSIR-JUNE-2012]

(a) $\frac{1}{2}xp^2 + V(x)$ (b) $\frac{\dot{x}^2}{2x} + V(x)$ (c) $\frac{1}{2}\dot{x}^2 + V(x)$ (d) $\frac{p^2}{2x} + V(x)$

5. The number of degrees of freedom of a rigid body in d space-dimensions is.

[CSIR-JUNE-2013]

(a) $2d$ (b) 6 (c) $d(d+1)/2$ (d) $d!$

6. The Lagrangian of a particle of mass m moving in one dimension is given by

$$L = \frac{1}{2}m\dot{x}^2 - bx$$

Where b is a positive constant. The coordinate of the particle x(t) at time t is given by: (in Following c_1 and c_2 are constants). **[CSIR-JUNE-2013]**

(a) $-\frac{b}{2m}t^2 + c_1 t + c_2$ (b) $c_1 t + c_2$

(c) $c_1 \cos\left(\frac{bt}{m}\right) + c_2 \sin\left(\frac{bt}{m}\right)$ (d) $c_1 \cosh\left(\frac{bt}{m}\right) + c_2 \sinh\left(\frac{bt}{m}\right)$

7. A particle moves in a potential $V = x^2 + y^2 + \frac{z^2}{2}$. Which component(s) of the angular momentum is/are constant(s) of motion? **[CSIR-DEC-2013]**

(a) None (b) L_x, L_y and L_z

(c) Only L_x and L_y (d) Only L_z

8. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T . The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be.

[CSIR-JUNE-2014]

(a) 0^0 to the vertical and $\sqrt{3}T$ (b) 30^0 to the vertical and T/2

(c) 60^0 to the vertical and $T/\sqrt{2}$ (d) 0^0 to the vertical and $T/\sqrt{3}$

9. The equation of motion of a system described by the time-dependent Lagrangian $L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right]$ is. **[CSIR-DEC-2014]**

(a) $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$ (b) $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$

(c) $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$ (d) $m\ddot{x} + \frac{dV}{dx} = 0$

10. If the Lagrangian of a dynamical system in two dimensions is $L = \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$, then its Hamiltonian is. **[CSIR-JUNE-2015]**

(a) $H = \frac{1}{m}p_x p_y + \frac{1}{2m}p_y^2$ (b) $H = \frac{1}{m}p_x p_y + \frac{1}{2m}p_x^2$

(c) $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_y^2$ (d) $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_x^2$

11. A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where k(t) is a time dependent parameter. Then $\frac{d}{dt}\langle V\rangle$, the rate of change of the expectation value $\langle V\rangle$ of the potential energy is. **[CSIR-JUNE-2015]**

(a) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{k}{2m}\langle xp + px\rangle$ (b) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle + \frac{1}{2m}\langle p^2\rangle$

(c) $\frac{k}{2m}\langle xp + px\rangle$ (b) $\frac{1}{2}\frac{dk}{dt}\langle x^2\rangle$

12. The Lagrangian of a system is given by. **[CSIR-DEC-2015]**

$L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 - 2q_2^2 - 2q_1q_2\right)$

where m and k are positive constants. The frequencies of its normal modes are.

(a) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{k}{2m}}\left(13 \pm \sqrt{73}\right)$

(c) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6k}{m}}$

13. The Lagrangian of a particle moving in a plane s given in Cartesian coordinates as

$$L = \dot{x}\dot{y} - x^2 - y^2$$

In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is. **[CSIR-JUNE-2016]**

(a) $\dot{r}\sin\theta + r\dot{\theta}\cos\theta$ (b) $\dot{r}\cos\theta + r\dot{\theta}\sin\theta$

(c) $2\dot{r}\cos\theta - r\dot{\theta}\sin 2\theta$ (d) $\dot{r}\sin 2\theta + r\dot{\theta}\cos 2\theta$

14. The Lagrangian of a system moving in three dimensions is. **[CSIR-JUNE-2016]**

$$L = \frac{1}{2}m\dot{x}_1^2 + m(\dot{x}_2^2 + \dot{x}_3^2) - \frac{1}{2}kx_1^2 - \frac{1}{2}k(x_2 + x_3)^2$$

The independent constants of motion is/are

(a) Energy alone

(b) Only energy, one component of the linear momentum and one component of the angular momentum

(c) Only energy, one component of the linear momentum

(d) Only energy, one component of the angular momentum

15. The dynamics of a particle governed by the Lagrangian

$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$ describes **[CSIR-DEC-2016]**

(a) An undamped simple harmonic oscillator

(b) A damped harmonic oscillator with a time varying damping factor

(c) An undamped harmonic oscillator with a time dependent frequency

(d) A free particle

16. The parabolic coordinates (ξ, η) are related to the Cartesian coordinates (x, y) by $x = \xi\eta$ and $y = \frac{1}{2}(\xi^2 - \eta^2)$. The Lagrangian of a two-dimensional simple harmonic oscillator of mass m and angular frequency ω is. **[CSIR-DEC-2016]**

(a) $\frac{1}{2}m\left[\dot{\xi}^2 + \dot{\eta}^2 - \omega^2(\xi^2 + \eta^2)\right]$

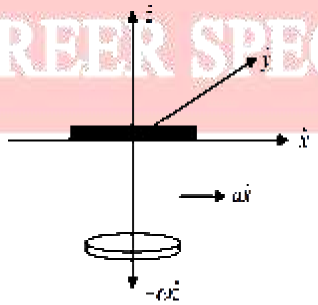
(b) $\frac{1}{2}m(\xi^2 + \eta^2)\left[(\dot{\xi}^2 + \dot{\eta}^2) - \frac{1}{4}\omega^2(\xi^2 + \eta^2)\right]$

(c) $\frac{1}{2}m(\xi^2 + \eta^2)\left[\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{2}\omega^2\xi\eta\right]$
(d) $\frac{1}{2}m(\xi^2 + \eta^2)\left[\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{4}\omega^2\right]$

17. The Lagrangian of a free relativistic particle (in one dimension) of mass m is given by $L = -m\sqrt{1-\dot{x}^2}$ where $\dot{x} = \frac{dx}{dt}$. If such a particle is acted upon by a constant force in the direction of its motion, the phase space trajectories obtained from the corresponding Hamiltonian are **[CSIR-JUNE-2017]**
(a) Ellipses (b) Cycloids (c) Hyperbolas (d) Parabolas

18. The energy of a one-dimensional system, governed by the Lagrangian
$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^{2n}$$
where k and n are two positive constants, is E_0. The time period of oscillation τ satisfies. **[CSIR-JUNE-2017]**
(a) $\tau \propto k^{-\frac{1}{n}}$ (b) $\tau \propto k^{-\frac{1}{2n}}E_0^{\frac{1-n}{2n}}$
(c) $\tau \propto k^{-\frac{1}{2n}}E_0^{\frac{n-2}{2n}}$ (d) $\tau \propto k^{-\frac{1}{n}}E_0^{\frac{1+n}{2n}}$

19. A disc of mass m is free to rotate in a plane parallel to the xy plane with an angular Velocity $-\omega\hat{z}$ about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction. The car accelerates in the positive x -direction with an acceleration $a >$ 0 . Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car? **[CSIR-DEC-2017]**



(a) Only the x and the z coordinates change
(b) Only the y and the z coordinates change
(c) Only the x and the y coordinates change
(d) All the three coordinates change

20. The motion of a particle in one dimension is described by the Langrangian $L = \frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2 - x^2\right)$ in suitable units. The value of the action along the classical path from $x = 0$ at $t = 0$ to $x = x_0$ at $t = t_0$, is. **[CSIR-DEC-2018]**

(a) $\frac{x_0^2}{2\,sin^2 t_0}$ (b) $\frac{1}{2}x_0^2$ tan t_0 (c) $\frac{1}{2}x_0^2$ cot t_0 (d) $\frac{x_0^2}{2\,cos^2 t_0}$

21. A frictionless horizontal circular table is spinning with a uniform angular velocity ω about the vertical axis through its centre. If a ball of radius a is placed on it at a distance r form the centre of the table. Its linear velocity will be.

[CSIR-NOV-2020]

(a) $-r\omega\hat{r} + a\omega\hat{\theta}$ (b) $r\omega\hat{r} + a\omega\hat{\theta}$

(c) $a\omega\hat{r} + r\omega\hat{\theta}$ (d) $-0(zero)$

HAMILTONIAN

1. The Hamiltonian of a system with n degrees of freedom is given $H(q_1, \dots q_n; p_1, \dots \dots . p_n; t)$, with an explicit dependence on the time t . Which of the following is correct? **[CSIR-JUNE-2011]**

(a) Different phase trajectories cannot intersect each other.

(b) H always represents the total energy of the system and is a constant of the motion.

(c) The equations $\dot{q}_1 = \frac{\partial H}{\partial p_i}, \dot{p}_1 = -\frac{\partial H}{\partial q_i}$ are not valid since H has explicit time dependence.

(d) Any initial volume element in phase space remains unchanged in magnitude under time evolution.

2. The Hamiltonian of a particle of unit mass moving in the xy -plane is given to be: $H = xp_x - yp_y - \frac{1}{2}x^2 + \frac{1}{2}y^2$ in suitable units. The initial values are given to be (x(0),y(0)=(1,1) and ($p_x(0)$, $p_y(0)$) = $\left(\frac{1}{2}, -\frac{1}{2}\right)$. During the motion, the curves traced out by the particles in the xy-plane and the p_xp_y – plane are. **[CSIR-JUNE-2011]**

(a) Both straight lines

(b) A straight line and a hyperbola respectively

(c) A hyperbola and an ellipse, respectively

(d) Both hyperbolas

3. The Hamiltonian of a simple pendulum consisting of a mass m attached to a massless string of length l is $H = \frac{p_\theta^2}{2ml^2} + mgl(1-\cos\theta)$. If L denotes the Lagrangian, the value of $\frac{dL}{dt}$ is: **[CSIR-DEC-2012]**

(a) $-\frac{2g}{l}p_\theta \sin\theta$ (b) $-\frac{g}{l}p_\theta \sin 2\theta$

(c) $\frac{g}{l}p_\theta \cos\theta$ (d) $lp_\theta^2 \cos\theta$

4. A system is governed by the Hamiltonian **[CSIR-JUNE-2013]**

$$H = \frac{1}{2}(p_x - ay)^2 + \frac{1}{2}(p_x - bx)^2$$

where a and b are constants and p_x, p_y are momenta conjugate to x and y respectively. For what values of a and b will the quantities $(p_x - 3y)$ and $(p_y + 2x)$ be conserved?

(a) a = -3,b = 2
(b) a = 3, b = -2
(c) a = 2,b = -3
(d) a = -2,b = 3

5. The Hamiltonian of a relativistic particle of rest mass m and momentum p is given by $H = \sqrt{p^2 + m^2} + V(x)$, in units in which the speed of light c =1. The corresponding Lagrangian is. **[CSIR-DEC-2013]**

(a) $L = m\sqrt{1 + \dot{x}^2} - V(x)$
(b) $L = -m\sqrt{1 - \dot{x}^2} - V(x)$
(c) $L = m\sqrt{1 + m\dot{x}^2} - V(x)$
(d) $L = \frac{1}{2}m\dot{x}^2 - V(x)$

6. A particle of mass m and coordinate q has the Lagrangian L $= \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$, where λ is a constant. The Hamiltonian for the system is given by. **[CSIR-JUNE-2014]**

(a) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2m^2}$
(b) $\frac{p^2}{2(m-\lambda q)}$
(c) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2(m-\lambda q)^2}$
(d) $\frac{p\dot{q}}{2}$

7. The Hamiltonian of a classical particle moving in one dimension is H $= \frac{p^2}{2m} + \alpha q^4$ where α is a positive constant and p and q are its momentum and position respectively. Given that its total energy $E \le E_0$ the available volume of phase space depends on E_0 as. **[CSIR-DEC-2014]**

(a) $E_0^{3/4}$
(b) E_0
(c) $\sqrt{E_0}$
(d) Is independent of E_0

8. The Hamiltonian of a system with generalized coordinate and momentum (q, p) is H = p^2q^2. A solution of the Hamiltonian equation of motion is (in the following A and B are constants). **[CSIR-JUNE-2016]**

(a) $p = Be^{-2At}, \quad q = \frac{A}{B}e^{2At}$
(b) $p = Ae^{-2At}, \quad q = \frac{A}{B}e^{-2At}$
(c) $p = Ae^{At}, \quad q = \frac{A}{B}e^{-At}$
(d) $p = 2Ae^{-A^2t}, \quad q = \frac{A}{B}e^{A^2t}$

9. The Hamiltonian for a system described by the generalized coordinate x and generalized momentum p is. **[CSIR-JUNE-2017]**

$$H = \alpha x^2 p + \frac{p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2$$

Where α, β and ω are constants. The corresponding Lagrangian is

(a) $\frac{1}{2}(\dot{x}-\alpha x^2)^2(1+2\beta x)-\frac{1}{2}\omega^2x^2$
(b) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2x^2-\alpha x^2\dot{x}$
(c) $\frac{1}{2}(\dot{x}-\alpha^2 x)^2(1+2\beta x)-\frac{1}{2}\omega^2x^2$
(d) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2x^2+\alpha x^2\dot{x}$

10. A Hamiltonian system is described by the canonical coordinate q and canonical momentum p. A new coordinate Q is defined as $Q(t)=q(t+\tau)+p(t+\tau)$, where t is the time and τ is a constant, that is, the new coordinate is a combination of the old coordinate and momentum at a shifted, that is, the new coordinate is a combination of the old coordinate and momentum at a shifted time. The new canonical momentum P(t) can be expressed as. **[CSIR-JUNE-2017]**
(a) $p(t+\tau)-q(t+\tau)$ (b) $p(t+\tau)-q(t-\tau)$
(c) $\frac{1}{2}[p(t-\tau)-q(t+\tau)]$ (d) $\frac{1}{2}[p(t+\tau)-q(t+\tau)]$

11. The Hamiltonian of a one-dimensional system is $H=\frac{xp^2}{2m}+\frac{1}{2}kx$, where m and k are positive constants. The corresponding Euler-Lagrange equation for the system is. **[CSIR-JUNE-2018]**
(a) $m\ddot{x}+k=0$ (b) $m\ddot{x}+2\dot{x}+kx^2=0$
(c) $2mx\ddot{x}-m\dot{x}^2+kx^2=0$ (d) $mx\ddot{x}+2m\dot{x}^2+kx^2=0$

12. The Hamiltonian of a classical one-dimensional harmonic oscillator is H = $\frac{1}{2}(p^2+x^2)$, in suitable units. The total time derivative of the dynamical variable $(p+\sqrt{2}x)$ is. **[CSIR-DEC-2018]**
(a) $\sqrt{2}p-x$ (b) $p-\sqrt{2}x$ (c) $p+\sqrt{2}x$ (d) $x+\sqrt{2}p$

POSSION BRACKET & CANONICAL TRANSFORMATION

1. The Poisson bracket $\{|\vec{r}|,|\vec{p}|\}$ has the value. **[CSIR-JUNE-2012]**
(a) $|\vec{r}||\vec{p}|$ (b) $\hat{r}.\hat{p}$ (c) 3 (d) 1

2. Let *A*, *B* and *C* be functions of phase space variables (coordinates and momenta of a mechanical system). If {,} represents the Poisson bracket, the value of {A, {B,C}}- {{A,B}, C}is given by. **[CSIR-DEC-2013]**
(a) 0 (b) {B, {C,A}} (c) {A, {C,B}} (d) {{C,A}, B}

3. The coordinates and momenta $x_i, p_i (i=1,2,3)$ of a particle satisfy the canonical Poisson bracket relations $\{x_i, p_j\}=\delta_{ij}$. If $C_1 = x_2p_3 + x_3p_2$ and C_2 = = $x_1p_2 + x_2p_1$ are constants of motion, and if $C_3 = \{C_1, C_2\} = x_1p_3 + x_3p_1$, then.

[CSIR-JUNE-2014]

(a) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = C_2$
(b) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = -C_2$
(c) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = C_2$
(d) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = -C_2$

4. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q = q^2$ and $P = p/2q$. The corresponding time independent generating function is. **[CSIR-JUNE-2014]**

(a) P/q^2 (b) q^2P (c) q^2/P (d) qP^2

5. A mechanical system is described by the Hamiltonian $H(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2q^2$. As a result of the canonical transformation generated by $F(q,Q) = -\frac{Q}{q}$, the Hamiltonian in the new coordinate Q and momentum P becomes. **[CSIR-DEC-2014]**

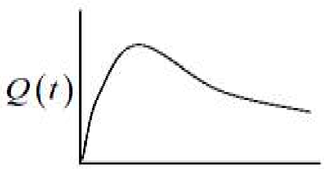
(a) $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}Q^2$ (b) $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}P^2$
(c) $\frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$ (d) $\frac{1}{2m}Q^2P^4 + \frac{m\omega^2}{2}P^{-2}$

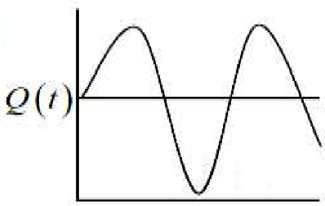
6. Let q and p be the canonical coordinate and momentum of a dynamical system. Which of the following transformations is canonical? **[CSIR-JUNE-2015]**

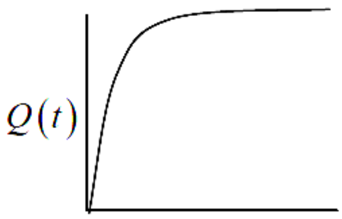
1. $Q_1 = \frac{1}{\sqrt{2}}q^2 \text{ and } P_1 = \frac{1}{\sqrt{2}}p^2$
2. $Q_2 = \frac{1}{\sqrt{2}}(p+q) \text{ and } P_2 = \frac{1}{\sqrt{2}}(p-q)$

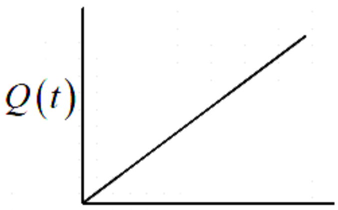
(a) Neither 1 nor 2 (b) both 1 and 2
(c) Only 1 (d) only 2

7. A canonical transformation (p,q) $\rightarrow (P,Q)$ is performed on the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2q^2$ via the generating function , $F = \frac{1}{2}m\omega q^2 \cot Q$. If Q(0), which of the following graphs shows schematically the dependence of $Q(t)$ on t?

(a)


(b)


(c)


(d)


8. Let (x,p) be the generalized coordinate and momentum of a Hamiltonian system.If new Variables (X,P) are defined by $X = x^{\alpha}$ sinh (βp) and $P = x^{\gamma}$ cosh(βp), where α, β and γ are constants, then the conditions for it to be a canonical transformation, are. **[CSIR-DEC-2017]**

(a) $\alpha = \frac{1}{2\beta}(\beta + 1)$ and $\gamma = \frac{1}{2\beta}(\beta - 1)$

(b) $\beta = \frac{1}{2\gamma}(\alpha + 1)$ and $\gamma = \frac{1}{2\alpha}(\alpha - 1)$

(a) $\alpha = \frac{1}{2\beta}(\beta - 1)$ and $\gamma = \frac{1}{2\beta}(\beta + 1)$

(a) $\beta = \frac{1}{2\gamma}(\alpha - 1)$ and $\gamma = \frac{1}{2\alpha}(\alpha + 1)$

Ans. : (c)

9. A canonical transformation $(q,p) \rightarrow (Q,P)$ is made through the generating function $F(q,P) = q^2 P$ on the Hamiltonian. $H(q,p) = \frac{p^2}{2\alpha q^2} + \frac{\beta}{4} q^4$. Where α and β are constants. The equations of motion for (Q, P) are. **[CSIR-JUNE-2016]**

(a) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\beta Q$

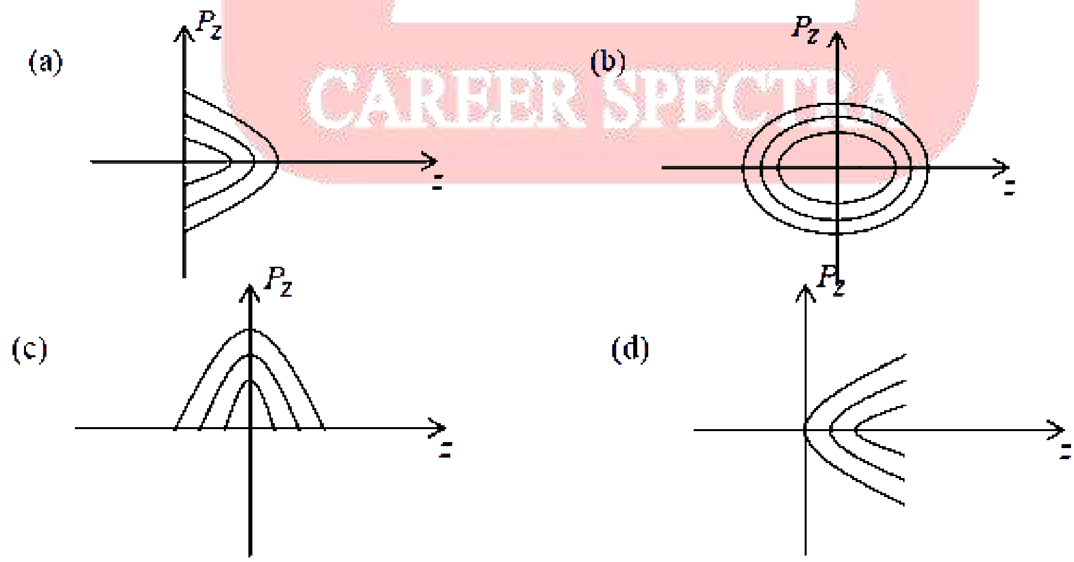
(b) $\dot{Q} = \frac{4P}{\alpha}$ and $\dot{P} = \frac{-\beta Q}{2}$

(c) $\dot{Q} = \frac{P}{\alpha}$ and $\dot{P} = -\frac{2P^2}{Q} - \beta Q$

(d) $\dot{Q} = \frac{2P}{\alpha}$ and $\dot{P} = -\beta Q$

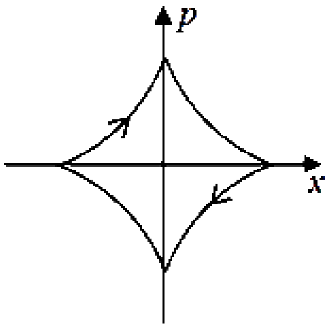
PHASE SPACE TRAJECTORY

1. The trajectory on the zp_z - plane (phase-space trajectory) of a ball bouncing perfectly elastically off a hard surface at $z = 0$ is given by approximately by (neglect friction): **[CSIR-DEC-2011]**

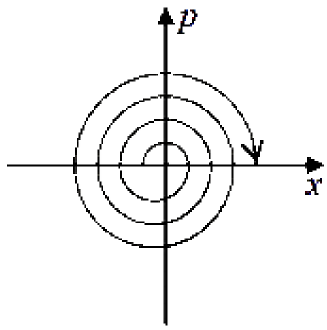


2. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum? **[CSIR-JUNE-2012]**

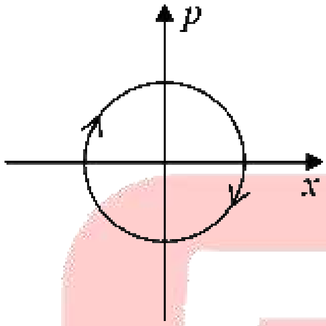
(a)



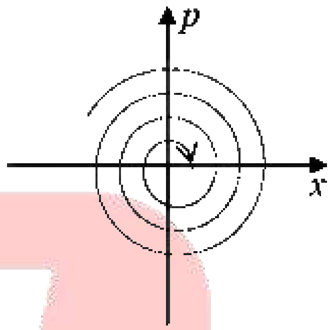
(b)



(c)

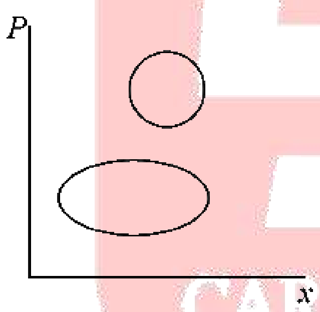


(d)

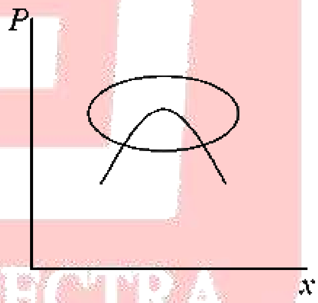


3. Which of the following set of phase-space trajectories is not possible for a particle obeying Hamilton's equations of motion? **[CSIR-DEC-2012]**

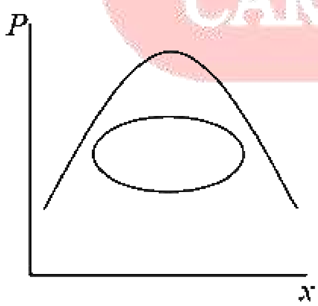
(a)



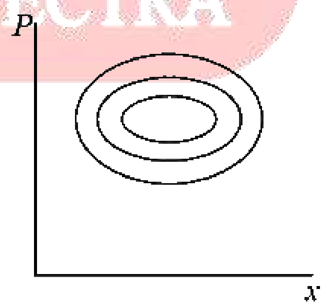
(b)



(c)

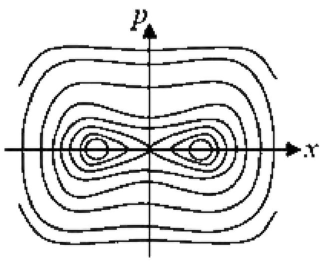


(d)

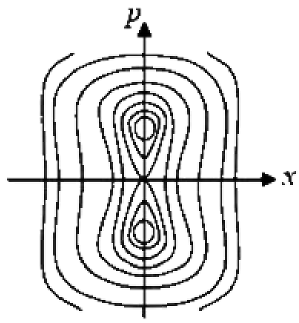


4. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional potential $V(x) = \frac{-1}{2}x^2 + \frac{1}{4}x^4$.

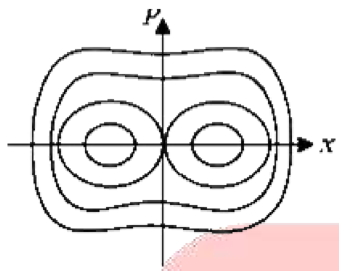
(a)



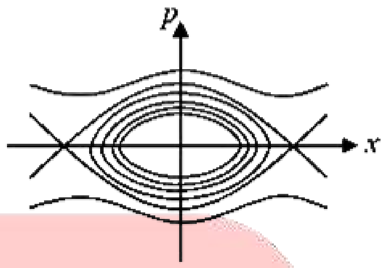
(b)



(c)

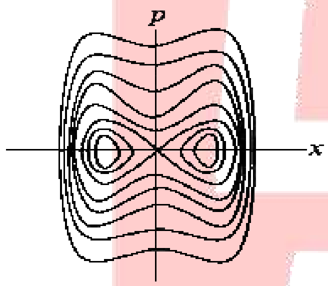


(d)

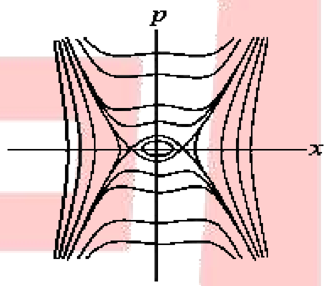


5. A particle moves in one dimension in a potential $V(x) = -k^2x^4 + \omega^2x^2$ where k and ω are constants. Which of the following curves best describes the trajectories of this system in phase space? **[CSIR-DEC-2017]**

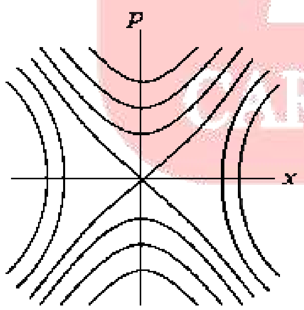
(a)



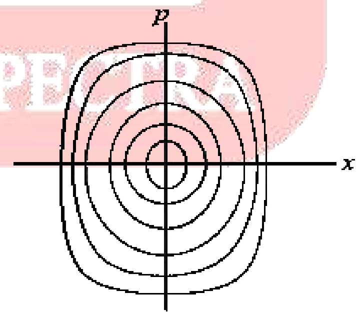
(b)



(c)

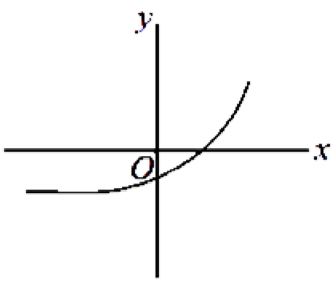


(d)

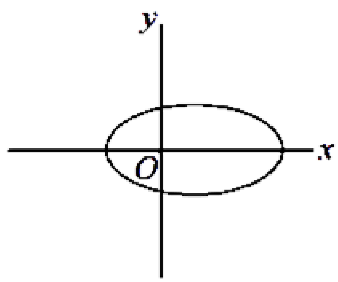


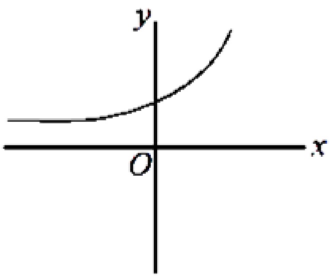
6. Which of the following figures best describes the trajectory of a particle moving in a repulsive central potential $V(r) = \frac{a}{r}(a > 0$ is a constant)?

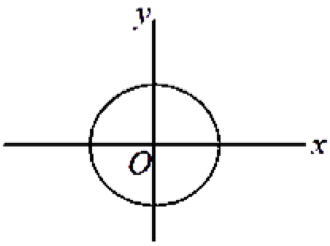
(a)



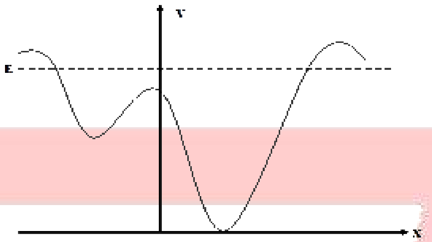
(b)



(c) 

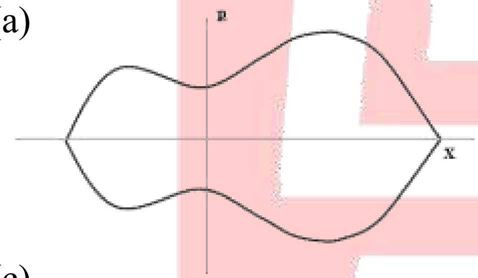
(d) 

7. Consider a particle with total energy E moving in one dimension n a potential V(x) as shown in the figure below: **[CSIR-NOV-2020]**

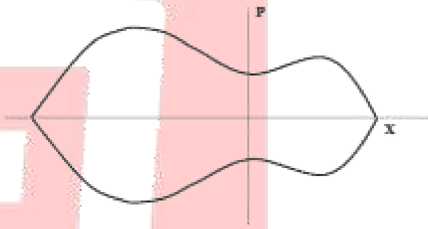


Which of the following figures best represents the orbit of the particle in the phase space?

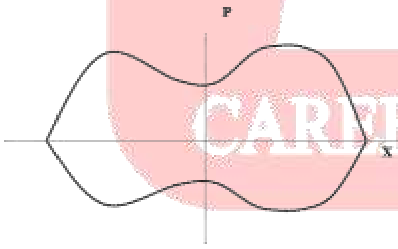
(a)



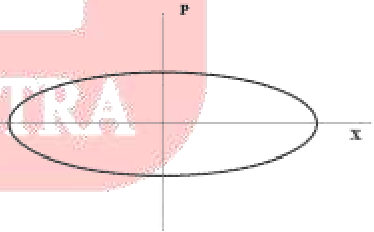
(b)



(c)



(d)



ANSWER-KEY

NEWTONIAN MECHANICS

1	B	2	D	3	A	4	B	5	D
6	C	7	B	8	C	9	A	10	B
11	C	12	A	13	B	14	A	15	B
16	C	17	B	18	B	19	A	20	D
21	B	22	A	23	C	24	A	25	C
26	B	27	C	28	C	29	*	30	*
31	*	32	*	33	*				

CENTRAL FORCE

1	C	2	D	3	A	4	A	5	D
6	B	7	B	8	C	9	C	10	*

SPECIAL THEORY OF RELATIVITY

1	B	2	D	3	B	4	B	5	D
6	A	7	C	8	C	9	B	10	None
11	D	12	D	13	A	14	A	15	C
16	B	17	B	18	B	19	C	20	B
21	C	22	A	23	*				

LAGRANGIAN

1	D	2	B	3	D	4	A	5	C
6	A	7	D	8	C	9	A	10	C
11	A	12	A	13	D	14	A	15	D
16	B	17	C	18	B	19	B	20	C
21	*								

HAMILTONIAN

1	A	2	D	3	A	4	D	5	B
6	B	7	A	8	A	9	A	10	D
11	C	12	A						

POSSION BRACKET & CANONICAL TRANSFORMATION

1	B	2	D	3	D	4	B	5	D
6	D	7	D	8	C	9	B		

PHASE SPACE TRAJECTORY

1	A	2	D	3	B	4	A	5	C
6	C	7	*						